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Bose–Einstein condensation at reheating

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We discuss the possibility that a perturbative reheating stage after inflation produces a scalar particle gas in a Bose condensate state, emphasizing the possible cosmological role of this phenomenon for symmetry restoration.

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I. INTRODUCTION

At the end of inflation, the Universe was in a very cold, low entropy phase. In particular, its energy density was still dominated by the vacuum energy of the fields which were previously driving inflation. The process of converting this energy into a hot thermal bath of matter and radiation is known as *reheating*, and it represents one of the most active areas of research in inflationary cosmology. In the last ten years, following the works [1,2], it has been shown that the first stages of reheating can be characterized by interesting nonperturbative phenomena, according to the particular theory one is considering. However, all these different possibilities share the common feature that they are followed by a more standard stage of perturbative particle production leading to the formation of the thermal bath. In the present work we focus on the possibility that after inflation some light boson field χ may experience a Bose-Einstein condensation, after production of the corresponding quanta via a perturbative reheating stage. This situation may be naturally realized if these quanta carry a charge which is either exactly conserved or weakly violated by interaction processes with all species produced at reheating. Condensation may then occur if a sufficiently large value for this charge is released during reheating .

The first condition is eventually a statement on the lagrangian density governing the χ field dynamics. As an example we consider a simple model of a complex scalar field χ whose interaction term is of the form of a quartic self-interaction monomial, invariant under a global $U(1)$ symmetry. Despite of its simplicity we look at this model as a prototype of much more complicated and realistic theories, as for example supersymmetric extensions of the electroweak standard model, which admit scalars as elementary excitations and conserved, or weakly violated, charges.

Specific mechanisms for production of a non zero and sufficiently large value for a definite charge at reheating have been widely discussed in the literature, aimed to drive, for example, efficient ways to produce baryon and lepton numbers. Among these, the Affleck-Dine scenario [3] has been shown to be able to produce very large baryon and lepton asymmetries. In this case scalar particles can naturally develop a Bose condensate.

In view of these considerations, we think that it is an interesting issue to study under which conditions a Bose condensate may actually form. Section II and III are devoted to this subject. In particular, in section II we report a fully general analysis of conditions necessary for the formation of the condensate, while in section III we discuss its actual formation in a cosmological context. We finally consider in section IV the possibility that the condensate may have an interesting cosmological implication, and leads to the phenomenon of symmetry restoration. This may reintroduce a monopole problem, since when symmetry eventually breaks again, once the expansion of the Universe dilutes the condensate, topological defects can be formed.

II. CONDITIONS FOR A CONDENSATE

In this section we discuss the conditions leading to the formation of a condensate. We consider a distribution of quanta of a light complex scalar field χ , whose mass m_χ is assumed to be dominated by radiative effects. We also assume that the initial distribution has a particle/antiparticle asymmetry, $Q \equiv n_\chi - n_{\bar{\chi}}$, and that the charge per comoving volume is conserved (or only very weakly violated) by the self-interactions among these quanta. * Without loss of generality, we consider $Q > 0$. The production of this charge will be discussed in the next section.

*We neglect here interactions with other fields; see the next section for a more general discussion of the system in a cosmological setup.

The evolution of the system is of course strictly dependent on these self-interactions, since they determine the thermalization time-scale as well as the radiative mass. As an example, let us consider the interaction term

$$\mathcal{L}_{\rm int} = -\,h\,|\chi|^4 \quad . \tag{1}$$

At lowest order in h, this term is responsible for $2 \leftrightarrow 2$ scatterings. Combining more vertices, processes which change the total number of particles $n = n_\chi + n_{\bar\chi}$ are possible, preserving however the charge Q. The efficiency of these processes in establishing a kinetic equilibrium, i.e. the thermalization time-scale, will be discussed in the next section. Here we consider the asymptotic equilibrium state in presence of a *fast* thermalization.

We start studying the case where a condensate does not form. The equilibrium distribution functions are therefore given by

$$f^{BE}_{\chi\,,\,\bar\chi}\,(\mathbf{p}) = \left[{\rm e}^{\,\beta(E\mp\mu)} - 1\right]^{-1} \quad . \tag{2}$$

In this expression, $E \equiv \sqrt{\mathbf{p}^2 + m_\chi^2(T)}$ denotes the energy of the quanta, while $\beta = 1/T$ is the inverse temperature [†]. The presence of the chemical potential μ is related to a nonvanishing charge Q. If Q and μ are both positive, the minus sign in Eq. (2) refers to particles χ, and the plus sign to antiparticles $\bar\chi$. The equality $\mu_\chi = -\,\mu_{\bar\chi} \equiv \mu$ holds in presence of effective number violating interactions such as $\chi\,\bar\chi\,\chi\,\bar\chi \leftrightarrow \chi\,\bar\chi$.

While the distribution function $f^{BE}_{\bar\chi}$ is always regular, we must impose $\mu < m_\chi$ to ensure the regularity of f^{BE}_{χ}. The values of β and μ can be determined by the two covariantly conserved quantities of the system, that is the total energy ρ and charge Q densities

$$\rho = \int \frac{d^3\mathbf{p}}{(2\,\pi)^3}\; E\;\left[f^{BE}_\chi\,(\mathbf{p}) + f^{BE}_{\bar\chi}\,(\mathbf{p})\right] \quad , \tag{3}$$

$$Q = \int \frac{d^3\mathbf{p}}{(2\,\pi)^3}\;\left[f^{BE}_\chi\,(\mathbf{p}) - f^{BE}_{\bar\chi}\,(\mathbf{p})\right] \quad . \tag{4}$$

Let us consider the ratio

$$R\,(\mu/T\,,\,m_\chi/T) \equiv \frac{Q}{\rho^{3/4}} \quad . \tag{5}$$

Notice that since we deal with radiation this ratio is unaffected by the universe expansion, since both quantities Q and $\rho^{3/4}$ scale as the inverse third power of the scale factor, so that their ratio R is constant. It is easy to see that R monotonically increases with μ/T. In particular, it ranges between 0 and the critical value $R_{cr}\,(m_\chi/T) \equiv R\,(m_\chi/T, m_\chi/T)$ as μ ranges between 0 and m_χ. We have thus a simple criterion to understand whether a condensate forms. From the initial conditions for the system at reheating we calculate the quantity $R = Q/\rho^{3/4}$, which is conserved by the interactions among the quanta χ and $\bar\chi$. If this value is below $R_{cr}\,(m_\chi/T)$, the equilibrium distributions are just of the form (2), that is the condensate does not form. When instead the initial conditions are such that R is greater than $R_{cr}\,(m_\chi/T)$, Eqs. (2) cannot represent the final distributions, but an additional singular term is present in $f_\chi(\mathbf{p})$

$$f_\chi\,(\mathbf{p}) = f^{BE}\,(\mathbf{p}) + (2\pi)^3 Q_c\,\delta^3\,(\mathbf{p}) \quad . \tag{6}$$

This shows the appearance of a condensate at zero momentum. The physical interpretation of this is very simple. Loosely speaking, $R > R_{cr}\,(m_\chi/T)$ corresponds to a case in which a too large charge Q is present to be stored in a regular distribution function with the given energy density ρ. The exceeding part of this charge, denoted by Q_c, gives rise to the condensate.

The value of $R_{cr}\,(m_\chi/T)$ can be obtained from expressions (3) and (4) in the limit $\mu \to m_\chi$ for the distribution (2). The calculation can be performed numerically, or analytically when m_χ/T is much smaller than one. In this regime we find

$$\rho = T^4\left(\frac{\pi^2}{15} + \mathcal{O}\left(\frac{m_\chi^2}{T^2}\right)\right) \quad , \tag{7}$$

$$Q = T^3\left(\frac{1}{3}\,\frac{m_\chi}{T} + \mathcal{O}\left(\frac{m_\chi^3}{T^3}\right)\right) \quad , \tag{8}$$

such that the scaling $R_{cr}\,(m_\chi/T) \simeq 0.456\,m_\chi/T$ is expected for $m_\chi/T \ll 1$. We plot in Figure 1 both the exact numerical behavior and the analytical approximation for $R_{cr}\,(m_\chi/T)$. As we see, the analytical approximation is very good even for m_χ as large as $0.1 \div 0.2\,T$.

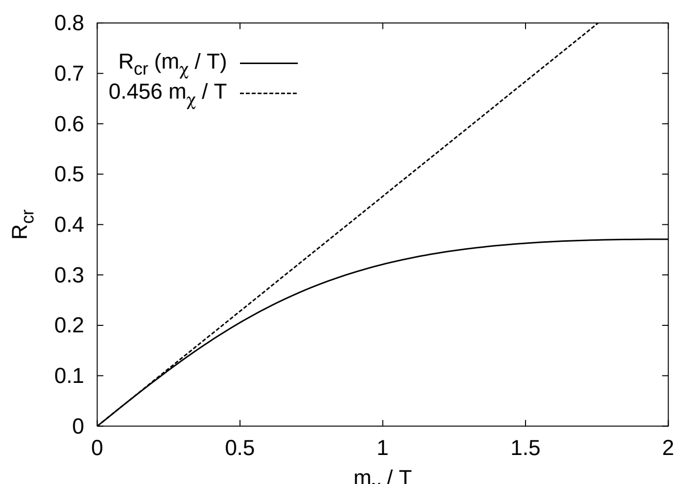

FIG. 1. The value of $R_{cr}\,(m_\chi/T)$ versus m_χ/T both numerically (solid line) and with an analytic approximation valid at small arguments (dotted line). Initial conditions such that $R > R_{cr}\,(m_\chi/T)$ indicate the formation of a condensate.

[†]We use natural units $c = \hbar = k_B = 1$.

If the field χ is very light, its mass will be dominated by radiative effects. In particular, from the interaction (1) among particles in the thermal distributions (2) we can perturbatively evaluate the thermal mass for $h \ll 1$. From one loop correction to χ propagator one easily find

$$m_\chi^2 = 4h \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E} \left(f_\chi^{BE}(\mathbf{p}) + f_{\bar{\chi}}^{BE}(\mathbf{p}) \right) \quad , \tag{9}$$

and hence

$$\frac{m_\chi(T)}{T} \sim \sqrt{\frac{h}{3}} \quad , \tag{10}$$

Therefore we see that a condensate forms whenever the initial conditions result in

$$R \gtrsim 0.26\ h^{1/2} \quad . \tag{11}$$

In the next sections we will discuss scalar condensation in a cosmological context. It is important to stress again that the expansion of the Universe does not modify the above condition (11), at least until the temperature drops below the value of the (non-thermal) mass of χ.

We now consider the case $R > R_{cr}(m_\chi/T)$, so that a condensate forms. In general with the distribution function for the scalars χ given by Eq. (6), both charge Q and energy density ρ are shared by the regular part $f_{\chi,\bar{\chi}}^{BE}$ and the condensate. The analysis greatly simplifies in the limit where most of the the charge Q is stored in the condensate. It is worth noticing that this is also the case we are more interested in. When a large charge is initially produced, the formation of the Bose condensate may lead to cosmologically relevant effects. In this case, we have $n_\chi \sim Q_c \gg n_{\bar{\chi}}$ and $Q_c \gg Q_{th}$, where we have defined with Q_c and $Q_{th} = Q - Q_c$ the charge accomodated in the condensate and in the thermal distribution respectively. Whenever number density is dominated by the condensate, evaluation of the thermal mass using Eq. (6) gives

$$\begin{aligned} m_\chi^2 \sim 4h \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E} \big(& f_\chi^{BE}(\mathbf{p}) + f_{\bar{\chi}}^{BE}(\mathbf{p}) \\ & + (2\pi)^3 Q_c \delta^3(\mathbf{p}) \big) \sim 2h \frac{Q_c}{m_\chi} \quad , \end{aligned} \tag{12}$$

and therefore

$$m_\chi(Q_c) \sim (2h\, Q_c)^{1/3} \, . \tag{13}$$

This result of course holds when $Q_c^{1/3} > T$. More generally, for smaller $Q_c^{1/3}/T$ it is easy to check that a very good approximation for the thermal mass is given by

$$m_\chi \sim \left(2hQ_c + \frac{1}{3} h T^3 \right)^{1/3} \quad . \tag{14}$$

The total number density can be estimated as follows

$$n \sim Q_c + n_{th}(Q_c, T) \quad , \tag{15}$$

where the thermal contribution $n_{th}(Q_c, T)$ can be numerically evaluated using the expression $f_{\chi,\bar{\chi}}^{BE}(\mathbf{p})$ with $\mu_\chi = m_\chi$, and m_χ given by Eq. (14). In particular when the temperature is larger than the effective mass, though it is still smaller than $Q_c^{1/3}$

$$(2hQ_c)^{1/3} < T < Q_c^{1/3} \quad , \tag{16}$$

we have

$$n_{th} \sim \frac{2\zeta(3)}{\pi^2} T^3 \quad . \tag{17}$$

Similarly the energy density can be written as

$$\rho \sim m_\chi Q_c + \rho_{th}(Q_c, T) \quad . \tag{18}$$

where $\rho_{th}(Q_c, T)$ and $m_\chi Q_c$ are the thermal contribution and the energy density stored in the condensate respectively.

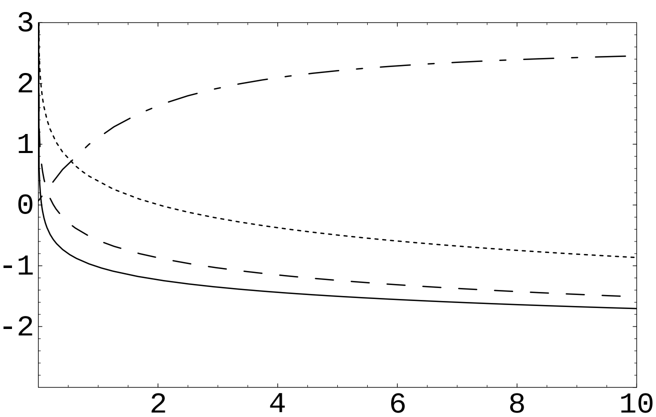

FIG. 2. The ratios Q_{th}/Q_c (solid line), n_{th}/Q_c (dashed line), $\rho_{th}/m_\chi Q_c$ (dotted line) and R (dash-dotted line), in logarithmic scale, versus Q_c/T^3. Q_{th} is the charge stored in the thermal distribution. Results are shown for $h = 0.01$.

In Figure 2 we plot the values for the ratios Q_{th}/Q_c, n_{th}/Q_c, $\rho_{th}/m_\chi Q_c$, and R, as functions of the variable Q_c/T^3, where Q_{th} is the charge stored in the thermal component. Results corresponds to $h = 0.01$ and have been evaluated using Eq. (14). Notice that for $Q_c \sim T^3$ the energy density is still dominate by the thermal distribution, while both the charge and number density receive their main contribution by the condensate.

Changing the value of h slightly affects all quantities, but it does not change the monotonic decreasing behaviour as function of Q_c/T^3 shown in Fig. 2. As an illustration we show in Fig. 3 how the above quantities depends on h, at $Q_c/T^3 = 1$, choosing h in the range $10^{-10} \div 10^{-1}$.

It is rather important to notice that the total energy density ρ always redshifts as the one of radiation. While this is obvious for the thermal part of the distribution, for the condensate it follows from the dependence of m_χ

on Q_c and T given in Eq. (14). As a consequence, the energy density stored in the condensate evolves in time as $Q_c^{4/3} \sim a^{-4}$, where a is the scale factor of the Universe.

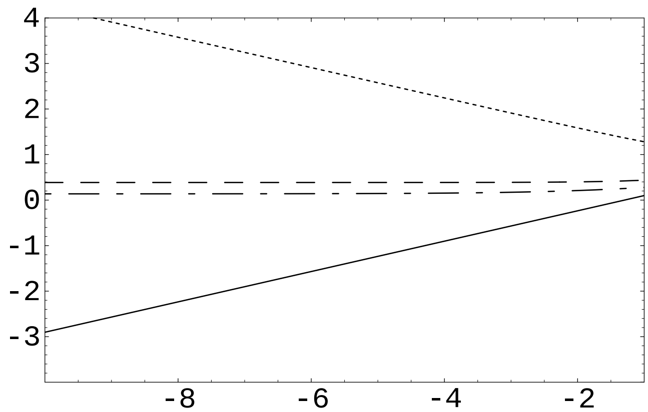

FIG. 3. The ratios Q_{th}/Q_c (solid line), n_{th}/Q_c (dashed line), $\rho_{th}/m_\chi Q_c$ (dotted line) and R (dash-dotted line), in logarithmic scale, versus *log h*, at fixed $Q_c/T^3 = 1$.

As we have said, we will be mainly interested in the following in studying the case where Q_c represents the dominant contribution to n. This condition is satisfied whenever $Q_c > n_{th}$. Using the behaviour of n_{th} and ρ with Q_c/T^3 this condition can be cast in a lower bound for the ratio $Q/\rho^{3/4}$

$$\frac{Q}{\rho^{3/4}} \gtrsim \frac{1}{3} \div \frac{1}{2} \quad , \tag{19}$$

as h varies in the range $10^{-10} \div 10^{-1}$. Condition (19) is of course more severe than Eq. (11), which merely states the formation of a condensate regardless of its magnitude with respect to n_{th}. The order of magnitude of this result, obtained by a numerical study, can be also achieved analitically by noticing that for all values of the coupling we have considered, when $n_{th} \sim Q_c$ the energy density receives the dominant contribution by the thermal part of the $\chi - \bar{\chi}$ distribution, so that

$$\rho \sim \rho_{th} = \frac{\pi^2}{15} T^4 \quad . \tag{20}$$

Using the fact that $\rho_{th}^{3/4} \sim 3 n_{th}$ (see Eq. (17)) we obtain that Q_c dominates the number density when $Q_c > n_{th} \sim \rho_{th}^{3/4}/3$, i.e. whenever $R \gtrsim 1/3$.

III. FORMATION OF THE CONDENSATE

In this section we discuss a possible scenario for the formation of a condensate in the early stages of the Universe. ‡. We start with a field ϕ which is oscillating around the minimum of its potential $V(\phi)$. § One interesting possibility is that ϕ is a modulus field of a supersymmetric theory. SUSY models offer several candidates for moduli fields, since their scalar potential almost unavoidably presents flat directions if supersymmetry is preserved. Most of these directions are expected to acquire a mass of the order of the supersymmetry breaking scale. Actually this scale, in the early Universe, can be very different from the present one, due to the dynamics of the inflaton or of other scalar fields during inflation and the first stages of reheating. See [7–9] for detailed discussions. Nevertheless, though supersymmetry naturally offers a theoretical framework, we aim to discuss the condensation phenomenon in a more general contest. For this reason we leave the mass m_ϕ as a free parameter. Also for the initial amplitude ϕ_0 of the field we have some natural scales, as for example the Planck mass, emerging in the context of supergravity models, or the breaking scale of GUT theories. However, also for ϕ_0 very different values can be considered in principle, and we will also take it as an input parameter.

At very early times the field ϕ is frozen at ϕ_0 , due to the friction provided by the expansion of the Universe. When, at a time $t \equiv t_0$, the expansion rate H falls down m_ϕ , the field ϕ starts oscillating around the minimum $\phi = 0$. The amplitude of these oscillations decreases as $a^{-3/2}$. If this stage is already radiation dominated, we then have

$$\phi\,(t) = \phi_0 \left(\frac{a_0}{a\,(t)}\right)^{3/2} \quad , \quad H\,(t) = m_\phi \left(\frac{a_0}{a\,(t)}\right)^2 \quad , \tag{21}$$

where $a_0 \equiv a\,(t_0)$.

If ϕ has a sufficiently long lifetime, its coherent oscillations will dominate over the thermal background for $a > a_{dom} \equiv a_0\,(3/8\pi)\,(M_{Pl}/\phi_0)^2$. After this time, the energy density stored in the oscillations redshifts with a matter-like behaviour, so that

$$H\,(t) = \sqrt{\frac{8\pi}{3}} m_\phi \frac{\phi_0}{M_{Pl}} \left(\frac{a_0}{a\,(t)}\right)^{3/2} \quad , \quad a > a_{dom} \quad , \tag{22}$$

while the amplitude $\phi\,(t)$ is still given by Eq. (21).

When eventually the ϕ field decays, it produces a reheating stage. In particular, baryon and lepton asymmetries can be generated in a rather natural way through the Affleck-Dine mechanism [3] if ϕ evolves along a complex direction and has baryon and lepton violating couplings. The same mechanism can be applied for the production of a generic charge Q . Let us indeed suppose that the field ϕ mainly decays at the time t_d into the complex scalar χ considered in the previous section. Through a

‡For related works, see [4–6].

§We assume that the minimum of the potential corresponds to $\phi = 0$ and that anharmonic terms in the Taylor expansion of $V(\phi)$ can be neglected, so that ϕ has a constant mass m_ϕ .

mechanism *à la* Affleck-Dine, these decays produce an asymmetry

$$Q \equiv n_\chi - n_{\bar{\chi}} = f\, n_\phi\left(t_d\right) \quad ,$$
$$n_\phi\left(t_d\right) \equiv \rho_\phi\left(t_d\right)/m_\phi = \phi_d^2\, m_\phi \quad . \tag{23}$$

Without entering into the details of the production, which is crucially dependent on the specific coupling between ϕ and χ and on the dynamics of the scalar field ϕ, we have parametrized with f in Eq. (23) the charge produced per quantum of ϕ. We have also parametrized with ϕ_d the amplitude of the oscillations of ϕ at decay.

It is immediate to see how large f should be in order to produce a condensate in the χ quanta distribution. Since $Q/\rho^{3/4} = f\, \rho_\phi\left(t_d\right)^{1/4}/m_\phi$, we see that the necessary condition for a condensate, Eq. (11), is met for

$$f^2 \gtrsim 0.07\, h\, \frac{m_\phi}{\phi_d} \quad . \tag{24}$$

The more stringent requirement that the number density of χ is dominated by the condensate, Eq. (19), rewrites instead

$$f^2 \gtrsim (0.1 \div 0.25)\, \frac{m_\phi}{\phi_d} \quad . \tag{25}$$

The interaction terms responsible for the decay of ϕ into χ will also give a mass to $\chi - \bar{\chi}$ quanta through radiative corrections. This mass vanishes once ϕ has decayed, so that the considerations of the previous section (for example, Eq. (14)) remain unchanged. However, it could prevent ϕ from decaying, whenever $m_\chi\left(\phi_d\right) > m_\phi/2$ (we do not consider here nonperturbative decay process, where this bound may not apply). This gives an upper bound on ϕ_d, which depends on the specific interaction terms between ϕ and χ. For interactions of the form $\sigma\phi\,\chi^2 + h.c.$, one finds for the decay rate

$$\Gamma\left(\phi \to \chi\chi\right) \simeq \frac{\sigma^2}{16\,\pi\, m_\phi^2}\, \sqrt{m_\phi^2 - 4\, m_\chi^2\left(\phi\right)} \quad ,$$
$$m_\chi^2(\phi) = \sigma\phi \tag{26}$$

which, equated to the expansion rate of the Universe, gives

$$\frac{\phi_d}{m_\phi} \simeq \frac{m_\phi}{\sigma}\left(-2r^2 + r\sqrt{1+4r^2}\right) \quad , \tag{27}$$

where we have defined

$$r \equiv \sqrt{\frac{3}{8\,\pi}\, \frac{\sigma^3 M_{Pl}}{16\pi m_\phi^4}} \sim 0.007\, \frac{\sigma^3\, M_{Pl}}{m_\phi^4} \quad . \tag{28}$$

We are assuming that the scalar ϕ decays after it dominates. Using Eq. (22), this translates in the bound

$$\phi_d < \phi_0\left(\frac{\phi_0}{M_{Pl}}\right)^3 \quad . \tag{29}$$

We show in Fig. 4 how the ratio ϕ_d/m_ϕ changes for different values of m_ϕ and σ. As it is clear from the above discussion, see for example Eq. (25), this is a crucial quantity, since an increase of ϕ_d over m_ϕ corresponds to an increase of the number of produced quanta over their energy. We first note that, for any fixed m_ϕ, there exists only a finite interval for σ where ϕ_d is enhanced. This is easily understood, since both a very low and a very high σ reduces the ϕ decay rate (26), so that when the decay eventually takes place the ϕ amplitude has been strongly damped by the expansion of the Universe. The decrease of $\Gamma(\phi \to \chi\chi)$ with decreasing σ is rather obvious, since σ measures the strength of the χ-ϕ coupling. In the limit of high σ, Eq. (27) reduces instead to $\phi_d/m_\phi \simeq m_\phi/\left(4\,\sigma\right)$. This shows that, in this limit, the ϕ particles are prevented from decaying by the high effective mass $m_\chi\left(\phi\right)$, and only once ϕ_d is sufficiently reduced the decay can occur.

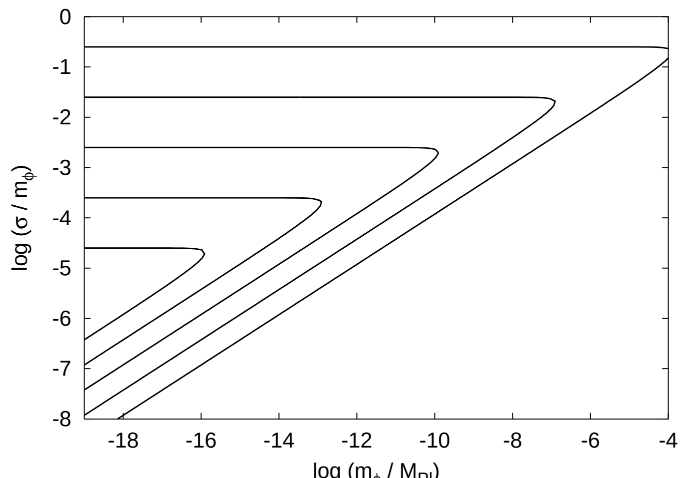

FIG. 4. Contours for the ratio ϕ_d/m_ϕ in the $log(m_\phi/M_{Pl})$ - $log(\sigma/m_\phi)$ plane. Continuous lines correspond, from right to left, to $\phi_d/m_\phi = 1, 10, 10^2, 10^3$ and 10^4.

For a given m_ϕ, the maximum of ϕ_d corresponds to $r = 1/\sqrt{5}$, that is for σ_* such that $\sigma_*/m_\phi \simeq 4\left(m_\phi/M_{Pl}\right)^{1/3}$. From Fig. 4 it is possible to understand the range for the coupling between the two species such that a sizeable condensate would appear. If σ is not too far from this value, we have from Eq. (27)

$$\frac{\phi_d}{m_\phi} \simeq 0.05\left(\frac{M_{Pl}}{m_\phi}\right)^{1/3} \quad , \tag{30}$$

that is ϕ_d/m_ϕ increases as m_ϕ decreases, as also clear from Fig. 4. Therefore, formation of a condensate is favoured at small m_ϕ. For σ close to the above value, the necessary condition Eq. (24), and the more stringent condition of Eq. (25) can indeed be cast in the form

$$f^2 \gtrsim 1.4\, h\left(\frac{m_\phi}{M_{Pl}}\right)^{1/3} \quad \text{and} \quad f^2 \gtrsim (2 \div 5)\left(\frac{m_\phi}{M_{Pl}}\right)^{1/3} \quad . \tag{31}$$

However, both the absolute value of ϕ_d and of $Q \simeq f\, m_\phi\, \phi^2$ decrease as m_ϕ becomes smaller. Hence, depending on which cosmologically scales one is interested in, the mass of the field ϕ cannot be taken arbitrarily small, if the condensate is supposed to play any relevant role. We will discuss this point in more details in the following section.

Until now we have simply assumed that, when all necessary conditions are fulfilled, the condensate forms due to a fast thermalization of the χ quanta. The actual generation of the condensate can be discussed by studying the typical timescales of self-interactions of the χ particles. [**]

Kinetic equilibrium among the quanta of χ is largely governed by $2 \leftrightarrow 2$ scatterings, allowed at tree level by the interaction term (1). At the initial stages, when the χ are produced by the ϕ field the corresponding rate can be estimated to be

$$\Gamma_{2\leftrightarrow 2} \sim h^2\, n/\langle E\rangle^2 \sim h^2\, \phi_d^2/m_\phi \quad , \tag{32}$$

where $n \geq Q$ is the number density of the particles in the initial state (initially $n \sim n_\phi$ given in Eq. (23)) and $\langle E\rangle$ their average energy, initially of the order of m_ϕ. However, a complete thermalization requires that also processes which violate the particle numbers are in equilibrium. Among these, the ones which appear at the lowest order in the coupling h are $4 \leftrightarrow 2$ processes, whose rates can be estimated as follows

$$\Gamma_{4\rightarrow 2} \sim h^4\, n^3/\langle E\rangle^8 \quad , \Gamma_{2\rightarrow 4} \sim h^4\, n/\langle E\rangle^2 \quad . \tag{33}$$

For a thermal distribution without a condensate, one has the simple relations $n \sim \langle E\rangle^3 \sim T^3$. However, we will always consider systems for which $n > \langle E\rangle^3$. In the initial configuration, this occurs for

$$\phi_d > m_\phi \quad . \tag{34}$$

Actually, we typically require $\phi_d \gg m_\phi$, i.e., as we have seen, small couplings $\sigma < m_\phi$, and $m_\phi \ll M_{Pl}$, see Eq. (27). The initial distribution of χ , $\bar\chi$ is thus characterized by high particle density and small energy of the individual quanta, so that the first stages of thermalization proceed via particle fusion. The decrease of n and increase of $\langle E\rangle$ have the general effect of decreasing the interaction rates. However, the condition $n > \langle E\rangle^3$ can be never violated if the system has a sufficiently high charge $Q > \rho^{3/4}$. This is equivalent to require that most of the particle in the final distribution will be stored in the condensate, Eq. (25). When this condition is met, all the interaction rates are thus always greater than the ones the χ, $\bar\chi$ would have *if they were* in a thermal distribution with the same energy $\rho_\chi \simeq m_\phi^2 \phi_d^2$, and without any condensate. If we therefore evaluate all relevant rates in the latter case, this will be a sufficient condition to ensure that the system we are considering rapidly approaches its equilibrium form.

Among the interactions we have considered, $2 \rightarrow 4$ processes have the lowest interaction rate. For a thermal distribution with temperature $T \sim \left(m_\phi^2\, \phi_d^2\right)^{1/4}$, we can estimate $\Gamma_{4\leftrightarrow 2} \sim h^4\, T$, so that thermalization is ensured, *a fortiori*, for

$$h \gtrsim h_* \equiv \left(\frac{\sqrt{m_\phi\, \phi_d}}{M_{Pl}}\right)^{1/4} \quad . \tag{35}$$

If the coupling h satisfies this bound, the χ , $\bar\chi$ system is expected to quickly thermalize right after it is formed at the decay time t_d, leading to the formation of the condensate, mainly populated by χ quanta, and a thermal distribution of both χ and $\bar\chi$.

For $h \ll h_*$ thermalization can occur only when the scale factor of the Universe becomes $a \sim a\,(t_d)\,(h_*/h)^4$, since the ratio Γ/H increases linearly with a. Therefore in this case the final effects of the condensate is expected to be strongly damped by the dilution of Q_c, due to the expansion of the Universe. The case $h \sim h_*$ requires more care, since (as we remarked) self-interactions between particles which are forming the condensate are much more efficient than in the thermal case ($n \gg \langle E\rangle^3$) and thus the formation of the condensate may be expected to hold in this case as well. However, we believe that a more precise answer in this regime cannot be obtained without a numerical study of the relevant Boltzmann equations. For this reason, hereafter the relation $h \gtrsim h_*$ will be always assumed.

Before concluding this section, it may be convenient to summarize the above results. We have considered some light scalars χ , $\bar\chi$ with an initial asymmetry Q. Eq. (24) is the necessary condition for a condensate to appear in the distribution of χ. The distributions for χ and $\bar\chi$ also keep a regular part (2), which contributes to the energy density of χ quanta. The total energy density redshifts as the one of radiation, as it is clear from Eqs. (18) and (14). The condition (25) corresponds to the case where most of the particle in the final distribution are stored in the condensate, so that the final number density approximatively coincides with Q_c. Of course this condition is a stronger requirement than just the formation of the condensate of Eq. (24). In the specific example we have discussed, the quanta of χ are produced by the decay of a massive scalar ϕ. We parametrized the energy density and the decay time of ϕ by its classical amplitude ϕ_d when it decays, given by Eq. (27). We have assumed that ϕ decays after it dominates, which is ensured by Eq. (29). Finally, (35) is a sufficient condition to guarantee fast self-interactions among the quanta of χ , $\bar\chi$, so that the condensate actually forms right after the decay of ϕ.

[**] For recent studies in thermalization, see [10–12].

IV. COSMOLOGICAL IMPLICATIONS

The aim of this section is to discuss the possible implications of the condensate of χ particles. In particular, we show that it may lead to symmetry restoration. As it is well known, this issue has crucial consequences for cosmology, since when symmetry eventually breaks again, once the expansion of the Universe dilutes the condensate, topological defects can be formed, reintroducing a monopole problem. The possibility of symmetry restoration after nonperturbative creation at *preheating* has been emphasized in several works [13–19]. Here we show that this may happen also in the ordinary, perturbative *reheating* stage, which is generally expected to occur in all models of inflation.

We suppose that the field χ is coupled to some other scalar field ψ. For definiteness we will consider ψ to be complex, though our result applies to a real scalar field as well. Besides this interaction, the field ψ is assumed to have the potential

$$V_\psi = \kappa \left(|\psi|^2 - \psi_0^2\right)^2 \quad , \tag{36}$$

and to be already settled down in a state with $|\langle\psi\rangle| = \psi_0$.

Due to the coupling to the ψ field, the scalar χ acquires in general an effective mass $m_\chi(\psi)$ and a modified dispersion relation, $\omega^2 = \mathbf{k}^2 + m_\chi^2(Q_c,T) + m_\chi^2(\psi)$, due to forward-scatterings, which however do not modify the distribution function of the χ quanta as long as $m_\chi(Q_c) \gg m_\chi(\psi)$. On the other hand this coupling also results in a contribution to the ψ one-loop effective potential [20,21],

$$\frac{\partial\, \Delta V_\psi}{\partial\, m_\chi^2(\psi)} = \frac{1}{(2\,\pi)^3} \int d^4p\, \delta\left(p^2 - m_\chi^2(Q_c,T) - m_\chi^2(\psi)\right) \cdot \left(f_\chi(\mathbf{p}) + f_{\bar\chi}(\mathbf{p})\right) \quad . \tag{37}$$

In case the χ number density is dominated by the condensate, $f_\chi(\mathbf{p}) \sim (2\pi)^3 Q_c \delta^3(\mathbf{p})$, this relation gives

$$\Delta V_\psi = \sqrt{m_\chi^2(Q_c,T) + m_\chi^2(\psi)}\, Q_c \quad , \tag{38}$$

where as discussed before we are considering $Q_c \sim n_\chi \gg n_{\bar\chi}$.

This term may shift the ground state of ψ to zero, thus restoring any symmetry which may be spontaneously broken when instead $|\langle\psi\rangle| = \psi_0$.

As an example, we consider an interaction term of the form $g^2\,|\psi|^2\,|\chi|^2$. In order to use the analysis contained in the previous sections, we first have to constrain our study to the case where the χ quanta mass is still dominated by the radiative term of Eq. (13), i.e.

$$g\psi_0 < (2hQ_c)^{1/3} \quad . \tag{39}$$

From Eq. (38) we therefore get

$$\Delta V_\psi \sim \left(m_\chi(Q_c,T) + \frac{g^2\psi^2}{2m_\chi(Q_c,T)}\right) Q_c \quad , \tag{40}$$

which shifts the minimum of the total potential $V_\psi + \Delta V_\psi$ to $\psi = 0$ for

$$g^2 Q_c \gtrsim 4\kappa\, m_\chi(Q_c)\, \psi_0^2 \quad . \tag{41}$$

The charge per unit volume Q_c takes its largest value at the decay of the ϕ field introduced in the previous section, and is then diluted by the expansion of the Universe. To see whether the symmetry broken by ψ is restored we evaluate the conditions (39) and (41) at the decay time t_d. Using the expression (23) for Q_c, we have

$$\frac{\psi_0}{(m_\phi\, \phi_d^2)^{1/3}} \lesssim \min\left[\frac{(2\,h\,f)^{1/3}}{g}\,,\, \frac{f^{1/3}\,g}{2^{7/6}\kappa^{1/2}h^{1/6}}\right] \quad . \tag{42}$$

For definiteness, and to get the largest scale which can be affected by the formation of the condensate, we consider the case where the ratio ϕ_d/m_ϕ is the largest compatible with Eq. (27), for each m_ϕ. This is equivalent to the choice σ_*/m_ϕ, so that from Eq. (30) we have

$$\frac{\psi_0}{m_\phi} \lesssim \left(\frac{M_{Pl}}{m_\phi}\right)^{2/9} \min\left[0.2\frac{(h\,f)^{1/3}}{g}\,,\, 0.06\frac{f^{1/3}\,g}{\kappa^{1/2}h^{1/6}}\right] \quad . \tag{43}$$

Once some specific values for the parameters are chosen, Eq. (43) indicates which is the maximal scale ψ_0 which can be restored by the condensate. It is not unconceivable that scales as high as the GUT one can be restored, provided m_ϕ is taken sufficiently high and the coupling σ is fixed to give the maximal ϕ_d as in Eq. (30). On the contrary, if we are interested in restoring a symmetry at a smaller scale, more freedom is allowed for m_ϕ, as well as for the value of ϕ_d, as it is clear from Eq. (42). We remind that these conditions must be supplemented by the ones found in the previous section, essentially Eq. (24), necessary condition for the condensate, and Eq. (35), implying a rapid thermalization of the system. It is worth noticing that these constraints are all mutually compatible.

V. CONCLUSIONS

In this paper we have considered the possibility that a light scalar field, whose quanta are excited at a perturbative reheating stage, may produce a large Bose-Einstein condensate. This phenomenon takes place if these quanta carry a conserved charge and the reheating stage produces large value for this charge. The motivation for this study relies on the fact that in scenarios as the Affleck-Dine models for baryon an lepton number generation, the decay of a modulus field may typically result in a quite large charge per comoving volume in the final relativistic degrees of freedom, and this charge is conserved or only weakly violated by the self-interaction processes after the reheating stage.

We have studied in details the conditions which should be fulfilled for the formation of the condensate for a complex scalar field with a simple quartic self-interaction term, and how the condensation process depends on the time scale for thermalization via scattering and number changing processes. For a wide range for the mass of the field whose decays lead to reheating and for the relevant coupling constants, the condensation phenomenon may easily occur and, moreover, the condensed particles can represent the main contribution to the total number per comoving volume.

We have finally considered the role that the condensate may have in restoring a symmetry which was broken at a larger energy scale. It is interesting to notice that also a grandunified symmetry, with scale $M_{GUT} \sim 10^{15} \div 10^{16}\ GeV$, can in principle be restored, although this requires some fine tuning in the parameters of the model. Restoration can occur more naturally for smaller scales, until the expansion of the Universe dilutes the charge per comoving volume which is stored in the condensate. This possibility would represent a new intriguing scenario in the evolution of the primordial Universe.

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[1] J.H. Traschen and R.H. Brandenberger, Phys. Rev. **D42**, 2491 (1990).
[2] L. Kofman, A. Linde, and A.A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994).
[3] I. Affleck and M. Dine, Nucl. Phys. **B249**, 361 (1985).
[4] D.V. Semikoz and I.I. Tkachev, Phys. Rev. **D55**, 489 (1997).
[5] D.V. Semikoz and I.I. Tkachev, Phys. Rev. Lett. **74**, 3093 (1995).
[6] D.V. Semikoz, Helv. Phys. Acta **69**, 207 (1996).
[7] G.F. Giudice, A. Riotto, and I. Tkachev, JHEP **9911**, 036 (1999).
[8] R. Kallosh, L. Kofman, A. Linde, and A. Van Proyen, Class. Quant. Grav. **17**, 4269 (2000).
[9] H. P. Nilles, M. Peloso, and L. Sorbo, hep-ph/0102264.
[10] S. Davidson and S. Sarkar, JHEP **0011**, 012 (2000).
[11] R. Allahverdi, Phys. Rev. **D62**, 063509 (2000).
[12] J. McDonald, Phys. Rev. **D61**, 083513 (2000).
[13] I.I. Tkachev, Phys. Lett. **B376**, 35 (1996).
[14] L. Kofman, A. Linde, and A.A. Starobinsky, Phys. Rev. Lett. **76**, 1011 (1996).
[15] E.W. Kolb and A. Riotto, Phys. Rev. **D55**, 3313 (1997).
[16] D. Boyanovsky, D. Cormier, H.J. de Vega, and R. Holman, Phys. Rev. **D55**, 3373 (1997).
[17] D. Cormier and R. Holman, Phys. Rev. **D60**, 041301 (1999).
[18] G. Felder and L. Kofman, Phys. Rev. **D63**, 103503 (2001).
[19] G. Felder, J. Garcia-Bellido, P.B. Greene, L. Kofman, A. Linde, and I.I. Tkachev, Phys. Rev. Lett. **87**, 011601 (2001).
[20] H. Umezawa, H. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States*, North-Holland 1982.
[21] P. Elmfors, K. Enqvist, and I. Vilja, Phys. Lett. **B326**, 37 (1994).